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Particle-Like Objects in a Nonlinear Field Theory

M. MURASKIN

Department of Physics, University of North Dakota, Grand Forks, North Dakota 58201

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Abstract

We further discuss the field theory which we introduced in a previous paper. We find that it is possible for a component of the field to have a minimum at an arbitrary origin point as a consequence of the field equations.

Considerable effort has gone into the search for elementary particles from an experimental approach. This effort has been rewarded by the discovery of an ever increasing number of particles. It remains clear that still higher energies will be needed before we gain an understanding of the structure of an electron which is treated, in present studies, as a point.

In atomic physics, spectroscopy has given us information concerning energy levels. Scattering experiments have also yielded important information. However, the shape of the electron distribution in the hydrogen atom in the various quantum states is most easily understood from a theoretical development—Schrödinger's equation. One would suspect that there should exist a partial differential equation from which we can describe the properties of elementary particles. In contrast to the experimental program, little effort has gone into the corresponding theoretical approach.

As a first step in this direction, we may seek a nonlinear field equation for which a field component has a maximum (minimum) at some point. Rosen (1966) has obtained a static solution to a nonlinear equation having the form

$$\theta = \frac{Z}{(Z^4 g + r^2)^{1/2}} \tag{1}$$

where Z and g are parameters. Born-Infeld (1934) found a solution of the type

$$E_r = \frac{q}{r_0{}^2} \frac{1}{\sqrt{[1 + (r/r_0)^4]}} \tag{2}$$

with q, r_0 as parameters. Anderson & Derrick (1970) have also studied particle-like behavior using a different set of nonlinear partial differential equations. These are some examples of the work being done. All these particle-like solutions are seen to have an overly simple type structure. At this stage, the problem of existence of particle-like behavior in nonlinear equations, we see, is a mathematical one, as there are no compelling physical arguments for the equations considered by these authors. Nevertheless, we feel that mathematical models of particles would be, as we said previously, a first step.

In previous papers (Muraskin, 1970a, b; Muraskin & Clark, 1970; Muraskin & Ring, 1971), we have introduced a field theory based on the Lorentz invariant equations

$$\frac{\partial \Gamma^i_{jk}}{\partial x^l} + \Gamma^m_{jk}\Gamma^i_{ml} - \Gamma^i_{mk}\Gamma^m_{jl} - \Gamma^i_{jm}\Gamma^m_{kl} = 0 \tag{3}$$

$$\frac{\partial g_{ik}}{\partial x^l} - \Gamma^m_{il} g_{mk} - \Gamma^m_{kl} g_{im} = 0 \tag{4}$$

In contrast to the equations of these other authors, the equations (3) and (4) can be motivated from simple ideas (Muraskin, 1970a). We have already obtained the following properties of these equations (Muraskin, 1970a, b; Muraskin & Clark, 1970; Muraskin & Ring, 1971):

(*a*) Nontrivial solutions to the equations exist locally.
(*b*) Equation (3) is related to the second derivative wave equation in an asymptotic sense.
(*c*) Dirac plane waves give an exact solution to the field equations. [This feature is not shared by all $e^\alpha{}_i$ plane waves. Here, we have $\Gamma^i_{jk} = e_\alpha{}^i(\partial e^\alpha{}_j/\partial x^k)$].
(*d*) An extremum in g (the determinant of g_{ik}) can be made to appear at an arbitrary origin.
(*e*) The field equations are capable of exhibiting a badly broken, yet well defined, reflection symmetry (as well as more complicated symmetries) about the point where g is an extremum.
(*f*) Many of the components of Γ^i_{jk} can have extremum behavior about the point where g is an extremum.
(*g*) For a choice of parameters at the origin point, we find that the simple equation

$$\Box g \equiv g^{ij}\frac{\partial^2}{\partial x^i}\frac{g}{\partial x^j} = 0$$

is contained in the field theory.

In this paper, we show that g_{00} can be made a minimum at an arbitrary origin point.

In our previous work, we introduced Γ^i_{jk} and $e^\alpha{}_i$ at the origin point such that

$$\Gamma^i_{jk} = e_\alpha{}^i e^\beta{}_j e^\gamma{}_k \Gamma^\alpha_{\beta\gamma} \tag{5}$$

$\Gamma^{\alpha}_{\beta\gamma}$ is chosen so that the integrability conditions

$$\Gamma^{\alpha}_{\beta\lambda}\Gamma^{\lambda}_{\gamma\rho} - \Gamma^{\alpha}_{\beta\lambda}\Gamma^{\lambda}_{\rho\gamma} + \Gamma^{\lambda}_{\beta\rho}\Gamma^{\alpha}_{\lambda\gamma} - \Gamma^{\lambda}_{\beta\gamma}\Gamma^{\alpha}_{\lambda\rho} = 0 \tag{6}$$

are satisfied.

From the field equations (4), we get

$$\frac{\partial g_{00}}{\partial x^k} = 2\Gamma^t_{0k} g_{0t} \tag{7}$$

For g_{00} to be an extremum, we have

$$\Gamma^t_{0a} g_{0t} = 0 \tag{8}$$

where $a = 1, 2, 3$. From (7) and the field equations for Γ^i_{jk}, and from (8) we get ($b = 1, 2, 3$)

$$A_{ba} \equiv \frac{\partial^2 g_{00}}{\partial x^b \partial x^a} = 2\{g_{0t}\Gamma^t_{00}\Gamma^0_{ab} + g_{0t}\Gamma^t_{ma}\Gamma^m_{0b} + g_{mt}\Gamma^t_{0a}\Gamma^m_{0b}\} \tag{9}$$

We note $A_{ba} = A_{ab}$ follows from (6). Taking the point P as the origin point, we have

$$g_{00}(Q) = g_{00}(P) + \frac{\partial g_{00}}{\partial x^a} dx^a + \frac{1}{2}\frac{\partial^2 g_{00}}{\partial x^a \partial x^b}\partial x^b \partial x^a + \cdots \tag{10}$$

Using (7) and (8), we get that $g_{00}(Q)$ is a maximum or minimum if $\frac{1}{2}A_{ba}dx^a dx^b$ always has the same sign for any point Q in the neighborhood of P. Thus, it follows that $A_{ba}dx^a dx^b$ must be positive or negative definite. Therefore, the conditions for a minimum are (Hildebrand, 1952) (g_{00} in our case will be negative)

$$A_{11} + A_{22} + A_{33} > 0 \tag{11a}$$

$$A_{11}A_{22} - (A_{12})^2 + A_{22}A_{33} - (A_{13})^2 + A_{33}A_{11} - (A_{13})^2 > 0 \tag{11b}$$

$$\begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix} > 0 \tag{11c}$$

In our previous work (Muraskin, 1970a; Muraskin & Clark, 1970; Muraskin & Ring, 1971), we used a particular choice of $\Gamma^{\alpha}_{\beta\gamma}$. A deficiency of this choice is as follows. We have, at the origin point, $2\Gamma^t_{tk} = \Gamma^t_{kt}$, Furthermore, it can be seen that this condition is maintained at all points by the field equations. Thus, we shall seek a less restrictive set of $\Gamma^{\alpha}_{\beta\gamma}$. We shall exhibit such a choice for $\Gamma^{\alpha}_{\beta\gamma}$ that satisfies (6) and for which (11) is also satisfied.

We shall take $\Gamma^{\alpha}_{\beta\gamma}$ to be zero if any of the indices take on the value zero, except for the case of Γ^0_{00} which is taken to be nonzero. We take† all

† No summation over repeated indices.

$\Gamma^a_{aa} = A$; all $\Gamma^a_{bc} = B$ where a, b, c are all different; all $\Gamma^a_{ac} = C$ for $c \neq a$; all $\Gamma^a_{ba} = D$ for $b \neq a$; all $\Gamma^a_{bb} = E$ for $a \neq b$. Then (6) goes into the following;

$$\begin{aligned} CD - AD - C^2 + AC + D^2 + BD - E^2 - EB &= 0 \\ BD - 2BC + AB + CD - BE &= 0 \\ 2ED - 2EC - D^2 + EA + AD - B^2 &= 0 \\ EC + 2BD - ED - AB - E^2 &= 0 \end{aligned} \tag{12}$$

A solution to the equations is given by

$$\begin{aligned} E &= 1 \\ D = B &= \text{arbitrary} \\ A &= 2B + 2 \\ C &= 3B + 1 \end{aligned} \tag{13}$$

The other solutions to these equations have been less useful for us. We have chosen

$$\begin{aligned} B = D &= -0{\cdot}7 \\ C &= -1{\cdot}1 \\ A &= 0{\cdot}6 \\ E &= 1 \\ \Gamma^0_{00} &= 16{\cdot}0 \end{aligned} \tag{14}$$

and

$$\begin{array}{llll} e^1{}_1 = 0{\cdot}8 & e^2{}_1 = 0{\cdot}9 & e^3{}_1 = 1{\cdot}0 & e^0{}_1 = 10{\cdot}2 \\ e^1{}_2 = -0{\cdot}3 & e^2{}_2 = -0{\cdot}4 & e^3{}_2 = -0{\cdot}15 & e^0{}_2 = 0{\cdot}08 \\ e^1{}_3 = -0{\cdot}2 & e^2{}_3 = -0{\cdot}25 & e^3{}_3 = -0{\cdot}35 & e^0{}_3 = 0{\cdot}09 \\ e^1{}_0 = 1{\cdot}6882 & e^2{}_0 = 2{\cdot}0570 & e^3{}_0 = 1{\cdot}8844 & e^0{}_0 = 2{\cdot}0 \end{array} \tag{15}$$

$e^1{}_0$, $e^2{}_0$, $e^3{}_0$ were obtained from (8), (15) and

$$g_{ij} = e^\alpha{}_i e^\beta{}_j g_{\alpha\beta} \tag{16}$$

with $g_{\alpha\beta} = (+1, -1, -1, -1)$. $e^1{}_0$, $e^2{}_0$, $e^3{}_0$ were obtained to 13 decimal places, but they are rounded off in (15). We then obtain

$$\begin{aligned} A_{11} + A_{22} + A_{33} &= 0{\cdot}64503 \times 10^5 \\ A_{11}A_{22} - (A_{12})^2 + A_{22}A_{33} - (A_{23})^2 + A_{11}A_{33} - (A_{13})^2 &= 0{\cdot}14234 \times 10^7 \\ \det A_{ab} &= 0{\cdot}19799 \times 10^6 \end{aligned} \tag{17}$$

This implies that g_{00} is a minimum at the origin point.

Thus, we conclude that the field equations are capable of describing a minimum for a field component. A maximum (minimum) in g rather than g_{00} has, at present, not been found.

With the help of a computer, we have mapped the solution in the neighborhood of the origin. In contrast to our previous papers, we find no obvious symmetries in this case, nor did we find any line-up of components. This points out the diversity of effects which the field equations are capable of describing.

We have not yet established whether these particle-like objects are bounded. We also do not know whether these field equations are consistent with many maxima and minima at different points in space.

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